Abstracts

Semisimplification for algebraic (super)groups

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(joint work with Maria Gorelik, Rainer Weissauer)

The quotient $Rep(G)$ of finite dimensional representations of an algebraic supergroup by the negligible morphisms is of the form $Rep(G^{red}, \varepsilon)$ where G^{red} is an affine supergroup scheme and $Rep(G^{red}, \varepsilon)$ is the full subcategory of representations in $Rep(G)$ such that their $\mathbb{Z}/2\mathbb{Z}$-gradation is given by the operation of $\varepsilon : \mathbb{Z}/2\mathbb{Z} \to G^{red}$ [2]. It is better to semisimplify instead the full monoidal subcategory $Rep(G)^I$ of direct summands in iterated tensor products of irreducible representations of $Rep(G)$. One major problem is the computation of the Picard group of the quotient category $Rep(G)^I/\mathcal{N} =: Rep(G_I^{red}, \varepsilon)$.

In [4] the authors determined the connected derived groups $G_{n|n}$ of the group $H_{n|n} = G_I^{red}$ in case $G = GL(n|n)$. These results are based on semisimplicity statements about the Duflo-Serganova functor $DS : Rep(GL(m|n)) \to Rep(GL(m-k|n-k))$ as proven in [3]. The DS functor gives rise to a tensor functor between the semisimplifications and allows for an inductive determination of the semisimplification.

The entire $GL(m|n)$-case, $m \geq n$, can be reduced to the $m = n$-case as shown in upcoming work of Heidersdorf and Weissauer. Indeed one gets $G_{m|n} \cong SL(m-n) \times G_{n|n}$. Crucial here are two ingredients: One can basically decompose an irreducible representation of non-vanishing superdimension into a $GL(m-n)$-part and a $GL(n|n)$-part; and the explicit computation of $GL(m|2)$-tensor products to get the induction started.

Parts of this picture are now emerging for the orthosymplectic superalgebra $\mathfrak{osp}(m|2n)$ as well. In joint work with Maria Gorelik [1] we proved the semisimplicty of the DS functor for $\mathfrak{osp}$ and OSp. More precisely DS sends any semisimple to a semisimple representation and satisfies some purity property. This result implies that the DS functor gives rise to a tensor functor between the semisimplifications, so that the inductive determination of the groups $H_{m|2n}$ should work for $\mathfrak{osp}(m|2n)$ and $OSp(m|2n)$ similarly to the $GL(m|n)$-case.

References

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[2] Heidersdorf, T., *On supergroups and their semisimplified representation categories*, Algebr. Represent. Theory 22, No. 4, 937-959 (2019),

[3] Heidersdorf, T. and Weissauer, R., *Cohomological tensor functors on representations of the General Linear Supergroup* to appear in Mem. Am. Math. Soc., ArXiv e-prints, 1406.0321, 2014.

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